P-wave Quarkonium Decays to Meson Pairs

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Abstract

The processes of p-wave Quarkonium exclusive decays to two mesons are investigated, in which the final state vector mesons with various polarizations are considered separately. In the calculation, the initial heavy quarkonia are treated in the framework of non-relativistic quantum chromodynamics, whereas for light mesons, the light cone distribution amplitudes up to twist-3 are employed. It turns out that the higher twist contribution is significant and provides a possible explanation for the observation of the hadron helicity selection rule violated processes $\chi_{c1} \to \phi\phi, \omega\omega$ by the BESIII collaboration in recently. We also evaluate the $\chi_{b1} \to J/\psi J/\psi$ process and find that its branching ratio is big enough to be measured at the B-factories.

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I. INTRODUCTION

As one of the most interesting fields of high energy physics, the study of heavy quarkonium plays an important role in understanding the configuration of hadrons and the non-perturbative behavior of QCD. Within the quarkonium physics, recently the exclusive double charmonium production gains much attention in both theory and experiment. The double charmonium production at the LHC experiment [1] provides an opportunity to test the prevailing effective theory of quarkonium physics, the non-relativistic chromodynamics(NRQCD) [2], in high energy, and the double charmonium production at B-factories [3] poses a challenge to the leading order(LO) perturbative QCD(pQCD) calculation [4–6], which stimulated a series of followup theoretical studies [7–11].

For the double quarkonium production, since generally the color-octet mechanism [2] deduced from the NRQCD formalism is not that crucial as in some other situations, for instance in the inclusive quarkonium hadroproduction, it is tempting to apply the light cone formalism to the study [9]. At the B-factory, although the center-of-mass energy is only about 10 GeV, not very high, it is still much larger than the charm quark mass. And hence, the charm quarks in the double charmonium production may move on the light cone before hadronization. This fact inspires one to investigate the double charmonium production processes at the B-factory energy in the light cone formalism, though the form of charmonium light cone wave functions are far from mature.

At the B-factory, there are a huge number of bottomnium states, and the bottomonium decays to charmonium pairs are now under investigation in experiment [12]. Theoretically, several calculations on this issue are performed in the framework of NRQCD [13–16], but only a few are carried on in light cone formalism. In Refs. [17–19] the light cone formalism was applied to calculate the process of bottomonium decays to double charmonium. The authors of [17, 19] found that higher twist terms were important and the results of $Br(\eta_b \to J/\psi + J/\psi)$ are much larger than the leading order NRQCD [14] prediction, which asks for further studies on quarkonium production and decays in light

cone framework.

In the literature, the p-wave charmonia, the χ_{cJ}, exclusive decays to light meson pairs have been investigated extensively, by virtue of NRQCD [13], the light cone formalism with perturbative QCD(pQCD) factorization approach [20], and also the meson loop technique [21], respectively. However, the corresponding study in bottomonium sector, i.e., the study on the p-wave bottomonia, the χ_{bJ}, exclusive decays to light meson pairs and double charmonia are very limited. In Refs. [13, 18], p-wave bottomonia decays to double J/ψ are evaluated in light cone formalism at leading order in twist expansion and in the framework of NRQCD, but the results from different approaches do not agree with each other.

In this work, we study various processes of p-wave heavy quarkonium exclusive decays to meson pairs (VV or PP), in which the light cone distribution amplitudes up to twist-3 are applied to the final states. Notice that neither relativistic correction in NRQCD, nor the twist-2 contribution in light cone formalism can explain well the BESIII experimental data of $\chi_{c1} \to \phi\phi$, $\omega\omega$ processes, which implies that the helicity selection rule is seriously violated. We tend to think that the twist-3 contribution is not merely a higher order correction, but rather significant in the description of these processes.

The rest of this paper is organized as follows. In Section II we present the strategy and formalism of the study; in Section III, we perform the numerical calculation and give some discussion on the results; Section IV is devoted to a summary and conclusions. For the sake of the readers convenience, some of the formulas used are given in the Appendix.

II. CALCULATION SCHEME DESCRIPTION AND FORMALISM

A. The initial heavy quarkonium

To begin the calculation, we first determine the character of initial heavy quarkonium via partons scattering process $Q\bar{Q} \to q\bar{q}\ +\ q\bar{q}$, where Q stands for the heavy quark b

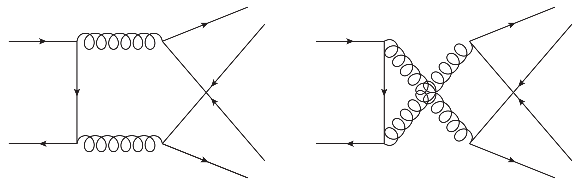

FIG. 1: Leading-order QCD Feynman diagrams for $\chi_{QJ} \to$ VV or PP process.

or c, and q for the constituent quarks of final state mesons. The schematic Feynman diagrams at the leading order in perturbative QCD are shown in Figure 1. For initial heavy quarks Q and $\bar{Q}$, their momenta are assigned as

$$p = \frac{P}{2} + l, \qquad \bar{p} = \frac{P}{2} - l, \tag{1}$$

respectively. Here, the relative momentum l between heavy quarks in the center-of-mass system(CMS) is much smaller than the total momentum P of the quark-antiquark system in the laboratory system(LS). To construct the spin-triplet states χ_{QJ}, one needs to project the colors and spinors of Q and $\bar{Q}$ to the proper quantum number of the states through projection operator by the standard approach in NRQCD [4, 22], that is

$$\Pi_3^\mu \epsilon_\mu \; = \; -\frac{(\not{p} + m_Q)(\not{P} + 2E_l)\gamma^\mu(\not{\bar{p}} - m_Q)}{4\sqrt{2}E_l(E_l + m_Q)}\epsilon_\mu \otimes \frac{\mathbf{1}_c}{\sqrt{N_c}} \; , \tag{2}$$

where $E_l^2 \; = \; P^2/4 \; = \; m_Q^2 - l^2$, $N_c \; = \; 3$, and $\mathbf{1}_c$ is the unit color matrix. The spin polarization vector ϵ_μ satisfies $P \cdot \epsilon = 0$ and $\epsilon \cdot \epsilon^* = -1$.

Generally, in the spin-triplet case the expansion of the matrix element in powers of l has the form [4]

$$\mathcal{M}[Q\bar{Q}(S=1)] \; = \; (\mathcal{A}_\rho + \mathcal{B}_{\rho\sigma} l^\sigma + \mathcal{C}_{\rho\sigma\tau} l^\sigma l^\tau + \ldots)\, \epsilon_S^\rho \; . \tag{3}$$

Then the matrix element for the spin-triplet P-wave heavy quarkonium states can be

expressed as:

$$\mathcal{M}[\chi_{Q0}] = \left(\frac{\langle O_1\rangle_{\chi_{Q0}}}{2N_c m_Q}\right)^{1/2} \mathcal{B}_{\rho\sigma}\, \frac{1}{\sqrt{3}}\, I^{\rho\sigma}\ , \tag{4a}$$

$$\mathcal{M}[\chi_{Q1}(\lambda)] = \left(\frac{\langle O_1\rangle_{\chi_{Q1}}}{2N_c m_Q}\right)^{1/2} \mathcal{B}_{\rho\sigma}\, \frac{i}{2m_Q\sqrt{2}}\, \epsilon^{\rho\sigma\lambda\mu} P_\lambda \epsilon_\mu(\lambda)\ , \tag{4b}$$

$$\mathcal{M}[\chi_{Q2}(\lambda)] = \left(\frac{\langle O_1\rangle_{\chi_{Q2}}}{2N_c m_Q}\right)^{1/2} \mathcal{B}_{\rho\sigma}\, \left[\frac{1}{2}(I^{\rho\mu}I^{\sigma\nu} + I^{\sigma\mu}I^{\rho\nu}) - \frac{1}{3}I^{\rho\sigma}I^{\mu\nu}\right] \epsilon_{\mu\nu}(\lambda)\ . \tag{4c}$$

Here, $\langle O_1\rangle_{\chi_{QJ}}$ are the NRQCD matrix elements; the tensor $I^{\mu\nu}$ reads

$$I^{\mu\nu} = -g^{\mu\nu} + \frac{P^\mu P^\nu}{4m_Q^2}\ , \tag{5}$$

and the sums of various polarizations of spin-1 polarization vector and spin-2 polarization tensors give

$$\sum_\lambda \epsilon^\mu(\lambda)\epsilon^\nu(\lambda) = I^{\mu\nu}, \tag{6a}$$

$$\sum_\lambda \epsilon^{\mu\nu}(\lambda)\epsilon^{\rho\sigma}(\lambda) = \frac{1}{2}(I^{\mu\rho}I^{\nu\sigma} + I^{\mu\sigma}I^{\nu\rho}) - \frac{1}{3}I^{\mu\nu}I^{\rho\sigma}\ . \tag{6b}$$

B. The light-cone distribution amplitudes

To calculate the hadron's distribution amplitude, the bi-spinors $v\bar{u}$ of the quark-antiquark pairs of final state mesons should be replaced by corresponding light-cone projectors. The light-cone projectors for light mesons can be readily obtained from Ref. [23], which is slightly different from what we used in the following by a factor of $(-i)$ in convention. The two-particle light-cone projection operator for a light pseudoscalar in momentum space up to twist-3 reads

$$M^P = \frac{-f_P}{4}\left\{ \not{p}'\gamma_5\,\phi(u) - \mu_P\gamma_5\left(\phi_\rho(u) - i\sigma_{\mu\nu}\, n_-^\mu n_+^\nu\, \frac{\phi'_\sigma(u)}{12} + i\sigma_{\mu\nu}p'^\mu\, \frac{\phi_\sigma(u)}{6}\, \frac{\partial}{\partial k_{\perp\nu}}\right)\right\}. \tag{7}$$

Here, p' stands for the momentum of meson; the parameters μ_P are $m_\pi^2/(m_u+m_d)$, $m_K^2/(m_u+m_s)$ and $m_{\eta_c}^2/2m_c^{\overline{MS}}$ in the $\overline{MS}$ scheme for pion, kion and η_c, respectively;

$\phi_\rho(u)$ and ϕ_σ are twist-3 distribution amplitudes; n_- and n_+ are light-like vectors satisfying $n_-^2 = 0$, $n_+^2 = 0$, $n_- \cdot n_+ = 2$; the component quarks' momenta are assigned as

$$k_1^\mu = uEn_-^\mu + k_\perp^\mu + \frac{\vec{k}_\perp^2}{4uE}\, n_+^\mu, \qquad k_2^\mu = \bar{u}En_-^\mu - k_\perp^\mu + \frac{\vec{k}_\perp^2}{4\bar{u}E}\, n_+^\mu \, . \tag{8}$$

Note that the term which involves the transverse momentum derivative acts on the hard scatter amplitude before collinear limit $k_1 = up' = uEn_-$ is taken. The asymptotic limit of the leading twist distribution amplitude takes the form $\phi(u) = 6u\bar{u}$. The two twist-3 distribution amplitudes ϕ_ρ and ϕ_σ can be obtained by solving the equations of motion and are determined as $\phi_\rho(u) = 1$ and $\phi_\sigma(u) = 6u\bar{u}$.

In the momentum space, the transverse and longitudinal light-cone projections of vector meson reads

$$\begin{aligned} M_\parallel^V \;=\; & \frac{f_V}{4}\,\frac{m_V(\varepsilon\cdot n_+)}{2E}\,E\,\not{n}_-\,\phi_\parallel(u) + \frac{f_V^T m_V}{4}\,\frac{m_V(\varepsilon\cdot n_+)}{2E}\Bigg\{ -\frac{i}{2}\,\sigma_{\mu\nu}\,n_-^\mu n_+^\nu\,h_\parallel^{(t)}(u) \\ & - iE\int_0^u dv\,(\phi_\perp(v) - h_\parallel^{(t)}(v))\;\sigma_{\mu\nu}n_-^\mu\,\frac{\partial}{\partial k_{\perp\nu}} + (1 - \frac{f_V(m_1+m_2)}{f_V^T m_V})\frac{h_\parallel'^{(s)}(u)}{2}\Bigg\}\Bigg|_{k=up'} \end{aligned} \tag{9}$$

and

$$\begin{aligned} M_\perp^V \;=\; & \frac{1}{4}f_V^T E\not{\varepsilon}_\perp\not{n}_-\phi_\perp(u) + \frac{1}{4}f_V m_V\left[\not{\varepsilon}_\perp g_\perp^{(v)}(u) - E\not{n}_-\int_0^u dv\left(\phi_\parallel(v) - g_\perp^{(v)}(v)\right)\varepsilon_\perp^\sigma\frac{\partial}{\partial k_\perp^\sigma}\right] \\ & + \frac{i}{4}\left(f_V - f_V^T\frac{m_1+m_2}{m_V}\right)m_V\varepsilon_{\mu\nu\rho\sigma}\varepsilon_\perp^\nu n_-^\rho\gamma^\mu\gamma_5\left(n_+^\sigma\frac{g_\perp'^{(a)}(u)}{8} - E\frac{g_\perp^{(a)}(u)}{4}\frac{\partial}{\partial k_{\perp\sigma}}\right)\Bigg|_{k=up'} \end{aligned} \tag{10}$$

respectively. Here, the transverse polarization vector

$$\varepsilon_\perp^\mu \equiv \varepsilon^\mu - \frac{\varepsilon\cdot n_+}{2}\,n_-^\mu - \frac{\varepsilon\cdot n_-}{2}\,n_+^\mu \, . \tag{11}$$

In the CMS of the initial quarkonium state, the light-cone vectors of final state mesons can be chosen as

$$n_- = (1,\ \sin\theta,\ 0,\ \cos\theta)\ , \qquad n_+ = (1,\ -\sin\theta,\ 0,\ -\cos\theta)\ . \tag{12}$$

For the vector meson moving along n_-, its polarization vectors are

$$\begin{aligned}\varepsilon(+) &= (0\,,-\frac{\cos\theta}{\sqrt{2}}\,,-\frac{i}{\sqrt{2}}\,,\frac{\sin\theta}{\sqrt{2}})\ ,\\ \varepsilon(-) &= (0,\frac{\cos\theta}{\sqrt{2}}\,,-\frac{i}{\sqrt{2}}\,,-\frac{\sin\theta}{\sqrt{2}})\ ,\\ \varepsilon(0) &= (\frac{\sqrt{E^2-m_V^2}}{m_V}\,,\frac{E\sin\theta}{m_V}\,,\ 0\,,\frac{E\cos\theta}{m_V})\ .\end{aligned} \tag{13}$$

The higher twist LCDAs are related to the leading twist ones through Wandzura-Wilczek relations [24]

$$g_\perp^{(v)}(u) = \frac{1}{2}\left[\int_0^u \frac{\phi_\parallel(v)}{\bar v}\,dv + \int_u^1 \frac{\phi_\parallel(v)}{v}\,dv\right], \tag{14}$$

$$g_\perp^{(a)}(u) = 2\left[\bar u\int_0^u \frac{\phi_\parallel(v)}{\bar v}\,dv + u\int_u^1 \frac{\phi_\parallel(v)}{v}\,dv\right], \tag{15}$$

$$h_\parallel^{(t)}(u) = (2u-1)\left[\int_0^u \frac{\phi_\perp(v)}{\bar v}\,dv - \int_u^1 \frac{\phi_\perp(v)}{v}\,dv\right], \tag{16}$$

$$h_\parallel^{(s)}(u) = 2\left[\bar u\int_0^u \frac{\phi_\perp(v)}{\bar v}\,dv + u\int_u^1 \frac{\phi_\perp(v)}{v}\,dv\right]. \tag{17}$$

Generally, while the above distribution amplitudes are convoluted with the hard part of a specific process, the troublesome endpoint divergences appears [25–27]. In this work the logarithmic and linear infrared divergences, appearing when the convolution integrations are performed, are attributed to certain complex quantities similar as the prescription used in Refs. [25–27], as

$$\int_0^1 \frac{du}{u} \to X_a\ , \qquad \int_0^1 du\,\frac{\ln u}{u} \to -\frac{1}{2}\,(X_a)^2\ , \qquad \int_0^1 \frac{du}{u^2} \to X_l - 1\ . \tag{18}$$

This measure is different from what in the pQCD approach taken in Ref.[20], where the Sudakov form factor is introduced in the convolution integration. Here, the first and the second are just the same as given in Ref. [26], and the third one should be less than 1 in magnitude, which is smaller than what in Ref. [26] because here the integral does not divergent at $u = 1$. We take the same assumption about X_a and X_l as in Ref. [26], i.e., they are universal to all final states with magnitudes around $\ln(m_Q/\Lambda_{QCD})$ and

m_Q/Λ_{QCD}, respectively. And, hence they can be constructed as

$$X_a = (1+\rho_a e^{i\phi_a})\ln(\frac{m_Q}{\Lambda_{QCD}}),\;\; X_l = (1+\rho_l e^{i\phi_l})\frac{m_Q}{\Lambda_{QCD}}\ . \tag{19}$$

The values of ρ and ϕ are such constrained that $\rho \leq 1$ and ϕ is arbitrary. In this work we take $\rho_a = \rho_l = 0.5$ and $\phi_a = \phi_l = 90^\circ$ by default, which is slightly different from what used in Refs. [26, 27] in B and B_s decays. The asymptotic amplitudes $\phi(u) = \phi_\sigma(u) = \phi_\perp(u) = \phi_\parallel(u) = 6u\bar{u}$ are taken in the calculation. To test the parameter dependence, we let ρ_l and ϕ_l to vary the same way as ρ_a and ϕ_a, respectively. Note that all these procedures in dealing with the divergences lead the calculation results to be model dependant.

C. The decay width

Once one knows the distribution amplitude, the decay widths can be readily obtained. The general polarized decay width reads:

$$\Gamma^J_{\lambda_1,\lambda_2} = \frac{1}{2M_{\chi_{QJ}}8\pi}\frac{1}{2J+1}\sqrt{1-\frac{4m^2}{M^2_{\chi_{QJ}}}}\int_{-1}^{1}\frac{d\cos\theta}{2}|\mathcal{M}^J_{\lambda_1,\lambda_2}|^2\ . \tag{20}$$

Here, m is the mass of light meson and J stands for the quantum number of total angular momentum. In case the final states are identical mesons, a statistical factor of $1/2$ should be added to the width (20). While final states being vector mesons, the total width then is:

$$\Gamma^J_{total} = \Gamma^J_{0,0} + 2\Gamma^J_{+,+} + 2\Gamma^J_{+,-} + 4\Gamma^J_{0,+}\ . \tag{21}$$

D. The calculation procedure

In our calculation, the computer algebra system **MATHEMATICA** is employed with the help of the packages **FEYNCALC** [28]. **FEYNCALC** is used to trace the Dirac matrices. The derivatives respect to the momentum are performed by means of the

MATHEMATICA code developed by ourself, and it has been tested before applying to the real calculation by recalculating the photonic decay rates of the χ_{c0} and χ_{c2} at the leading order and getting an agreement with Ref. [4]. The final analytical results are a little bit too lengthy to be listed here. The analytical expressions for $\mathcal{B}_{\rho\sigma}$ in equation (4) for various processes with different final states (VV or PP) are presented in the appendix for the sake of the readers convenience while they use.

III. NUMERICAL CALCULATION AND DISCUSSION

A. Input parameters

Before carrying out numerical calculations, the input parameters need to be fixed. As for the NRQCD matrix elements, we take those used in Refs. [29–32],

$$\begin{aligned}
\frac{1}{3}|\langle\chi_{c0}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\sigma)\chi|0\rangle|^2 &= 0.051\ \mathrm{GeV}^5\ , \\
\frac{1}{2}|\langle\chi_{c1}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\times\sigma\cdot\epsilon_H)\chi|0\rangle|^2 &= 0.060\ \mathrm{GeV}^5\ , \\
|\sum_{ij}\langle\chi_{c2}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{D}^{(i}\sigma^{j)}\epsilon_H^{ij})\chi|0\rangle|^2 &= 0.068\ \mathrm{GeV}^5\ , \\
\frac{1}{3}|\langle\chi_{b0}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\sigma)\chi|0\rangle|^2 &= 2.03\ \mathrm{GeV}^5\ , \\
\frac{1}{2}|\langle\chi_{b1}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\times\sigma\cdot\epsilon_H)\chi|0\rangle|^2 &= 2.03\ \mathrm{GeV}^5\ , \\
|\sum_{ij}\langle\chi_{b2}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{D}^{(i}\sigma^{j)}\epsilon_H^{ij})\chi|0\rangle|^2 &= 2.03\ \mathrm{GeV}^5\ .
\end{aligned} \tag{22}$$

The leading order running coupling constant

$$\alpha_s(\mu) = \frac{4\pi}{b_0\ln(\mu^2/\Lambda_{\mathrm{QCD}}^2)}$$

is employed with $b_0 = \frac{33-2n_f}{3}$. In numerical evaluation, we take the quark flavor number $n_f = 4$ and $\Lambda_{QCD} = 0.225$GeV; the interaction scale $\mu = 2m_b$ for the bottomonium decay processes, and $\mu = 2m_c$ for the charmonium decay processes; the up and down quarks are taken to be massless, while the strange quark mass $m_s = 0.10$GeV, the

charm quark mass $m_c = 1.4 \pm 0.2$GeV in NRQCD calculation, and the bottom quark mass $m_b = 4.8 \pm 0.1$GeV; the masses of quarkonia are obtained from the PDG [33], which are $M_{\chi_{b0}} = 9.859$GeV, $M_{\chi_{b1}} = 9.892$GeV, $M_{\chi_{b2}} = 9.912$GeV, $M_{\chi_{c0}} = 3.414$GeV, $M_{\chi_{c1}} = 3.510$GeV, and $M_{\chi_{c2}} = 3.556$GeV .

The input parameters for final state vector and pseudoscalar mesons are presented in Table I, which are obtained from Refs.[18, 19, 33]. For η_c, the charm quark mass in the $\overline{MS}$ scheme is taken to be $m_c^{\overline{MS}} = 1.2$GeV in light cone calculation [19].

TABLE I: Input parameters for final state mesons.

	ρ	$\bar{K}^*$	ω	ϕ	J/ψ
m_V[MeV]	770	892	782	1020	3097
f_V[MeV]	205 ± 9	217 ± 5	195 ± 3	231 ± 4	416 ± 5
f_V^T[MeV]	160 ± 10	170 ± 10	145 ± 10	200 ± 10	379 ± 21
	$\pi^+(\pi^-)$	$K^+(K^-)$	η_c		
m_P[MeV]	139.6	493.7	2980		
f_P[MeV]	130.4 ± 0.2	156.1 ± 0.8	373 ± 64		
μ_P[MeV]	1500	1700	3700		

B. Numerical results and Discussions

By virtue of the formulae provided in section II and combined with tensors given in appendix, one can readily get the analytical decay widths for final state mesons with specified helicity. After substituting the input parameters in preceding section to the analytical expressions, the numerical results are obtained. The magnitudes of decay widths for various processes, including helicity decay widths for final state vector mesons, are presented in Tables II to IV.

In our calculation, the uncertainties are estimated as follows. The first one comes from the uncertainty of heavy quark mass m_Q. The uncertainty in $\alpha_s(2m_Q)$ induced

TABLE II: The polarized decay widths for $\chi_{bJ} \to VV$ in unit of eV.

	$\Gamma_{0,\,0}$[eV]	$\Gamma_{+,\,+}$[eV]	$\Gamma_{+,\,0}$[eV]	$\Gamma_{+,\,-}$[eV]
$\chi_{b0} \to \rho^0\rho^0$	$2.60^{+0.48+0.09}_{-0.39-0.09}$	$0.009^{+0.003+0.079}_{-0.002-0.009}$	—	—
$\chi_{b0} \to \rho^+\rho^-$	$5.20^{+0.96+0.18}_{-0.78-0.18}$	$0.018^{+0.006+0.158}_{-0.004-0.018}$	—	—
$\chi_{b0} \to K^*\bar{K}^*$	$6.35^{+0.12+0.38}_{-0.10-0.14}$	$0.034^{+0.010+0.315}_{-0.008-0.034}$	—	—
$\chi_{b0} \to K^{*+}K^{*-}$	$6.35^{+0.12+0.38}_{-0.10-0.14}$	$0.034^{+0.010+0.315}_{-0.008-0.034}$	—	—
$\chi_{b0} \to \omega\omega$	$2.13^{+0.39+0.07}_{-0.32-0.07}$	$0.007^{+0.002+0.068}_{-0.002-0.007}$	—	—
$\chi_{b0} \to \phi\phi$	$3.93^{+0.71+0.41}_{-0.59-0.15}$	$0.034^{+0.010+0.306}_{-0.007-0.034}$	—	—
$\chi_{b0} \to J/\psi J/\psi$	$10.76^{+1.21+21.35}_{-1.10-6.79}$	$11.15^{+3.22+93.96}_{-2.46-10.89}$	—	—
$\chi_{b1} \to \rho^0\rho^0$	—	—	$1.2^{+0.3+25.9}_{-0.2-1.2} \times 10^{-3}$	—
$\chi_{b1} \to \rho^+\rho^-$	—	—	$2.4^{+0.6+51.8}_{-0.4-2.4} \times 10^{-3}$	—
$\chi_{b1} \to K^*\bar{K}^*$	—	—	$5.9^{+1.4+124.6}_{-1.1-5.9} \times 10^{-3}$	—
$\chi_{b1} \to K^{*+}K^{*-}$	—	—	$5.9^{+1.4+124.6}_{-1.1-5.9} \times 10^{-3}$	—
$\chi_{b1} \to \omega\omega$	—	—	$1.2^{+0.3+24.9}_{-0.2-1.1} \times 10^{-3}$	—
$\chi_{b1} \to \phi\phi$	—	—	$3.3^{+0.8+75.7}_{-0.6-3.3} \times 10^{-3}$	—
$\chi_{b1} \to J/\psi J/\psi$	—	—	$0.51^{+0.11+10.58}_{-0.09-0.25}$	—
$\chi_{b2} \to \rho^0\rho^0$	$0.14^{+0.03+0.03}_{-0.02-0.03}$	$0.50^{+0.14+6.80}_{-0.11-0.41} \times 10^{-3}$	$0.024^{+0.006+0.145}_{-0.005-0.023}$	$0.61^{+0.12+0.17}_{-0.10-0.10}$
$\chi_{b2} \to \rho^+\rho^-$	$0.28^{+0.06+0.06}_{-0.04-0.06}$	$1.0^{+0.28+13.60}_{-0.22-0.82} \times 10^{-3}$	$0.048^{+0.012+0.290}_{-0.010-0.046}$	$1.22^{+0.24+0.34}_{-0.20-0.20}$
$\chi_{b2} \to K^*\bar{K}^*$	$0.36^{+0.07+0.10}_{-0.05-0.09}$	$2.24^{+0.65+30.63}_{-0.50-1.80} \times 10^{-3}$	$0.082^{+0.019+0.485}_{-0.015-0.077}$	$1.65^{+0.33+0.59}_{-0.26-0.34}$
$\chi_{b2} \to K^{*+}K^{*-}$	$0.36^{+0.07+0.10}_{-0.05-0.09}$	$2.24^{+0.65+30.63}_{-0.50-1.80} \times 10^{-3}$	$0.082^{+0.019+0.485}_{-0.015-0.077}$	$1.65^{+0.33+0.59}_{-0.26-0.34}$
$\chi_{b2} \to \omega\omega$	$0.18^{+0.03+0.04}_{-0.03-0.03}$	$0.66^{+0.19+9.1}_{-0.15-0.59} \times 10^{-3}$	$0.032^{+0.007+0.188}_{-0.006-0.030}$	$0.78^{+0.15+0.22}_{-0.12-0.13}$
$\chi_{b2} \to \phi\phi$	$0.24^{+0.04+0.09}_{-0.04-0.09}$	$2.78^{+0.80+30.10}_{-0.61-2.71} \times 10^{-3}$	$0.077^{+0.018+0.474}_{-0.014-0.076}$	$1.57^{+0.31+0.65}_{-0.25-0.32}$
$\chi_{b2} \to J/\psi J/\psi$	$3.81^{+0.77+4.84}_{-0.63-3.81}$	$2.52^{+0.73+13.86}_{-0.56-2.14}$	$5.46^{+1.19+36.39}_{-0.97-4.94}$	$51.91^{+12.46+140.26}_{-9.85-33.66}$

by m_Q has also been taken into account in the evaluation. The second one comes from the uncertainties of parameters ρ and ϕ, and is evaluated by taking the decay width as function of ρ and ϕ. These two sources of uncertainty are the major ones in the calculation of this work.

TABLE III: The polarized decay widths for $\chi_{cJ} \to VV$ in unit of keV.

	$\Gamma_{0,\,0}$[keV]	$\Gamma_{+,\,+}$[keV]	$\Gamma_{+,\,0}$[keV]	$\Gamma_{+,\,-}$[keV]
$\chi_{c0} \to \rho^0\rho^0$	$3.96^{+9.63+0.99}_{-2.62-0.90}$	$0.31^{+1.84+2.81}_{-0.25-0.31}$	—	—
$\chi_{c0} \to \rho^+\rho^-$	$7.92^{+19.26+1.98}_{-5.24-1.80}$	$0.62^{+3.68+5.62}_{-0.50-0.62}$	—	—
$\chi_{c0} \to K^*\bar{K}^*$	$8.67^{+19.17+2.89}_{-5.66-2.56}$	$1.32^{+7.93+12.16}_{-1.07-1.32}$	—	—
$\chi_{c0} \to K^{*+}K^{*-}$	$8.67^{+19.17+2.89}_{-5.66-2.56}$	$1.32^{+7.93+12.16}_{-1.07-1.32}$	—	—
$\chi_{c0} \to \omega\omega$	$3.16^{+7.58+0.74}_{-2.08-0.68}$	$0.26^{+1.59+2.71}_{-0.22-0.26}$	—	—
$\chi_{c0} \to \phi\phi$	$4.21^{+7.27+2.36}_{-2.64-0.71}$	$1.09^{+6.56+9.95}_{-0.89-1.08}$	—	—
$\chi_{c1} \to \rho^0\rho^0$	—	—	$0.037^{+0.14+0.16}_{-0.028-0.030}$	—
$\chi_{c1} \to \rho^+\rho^-$	—	—	$0.074^{+0.28+0.32}_{-0.056-0.060}$	—
$\chi_{c1} \to K^*\bar{K}^*$	—	—	$0.097^{+0.35+0.43}_{-0.72-0.074}$	—
$\chi_{c1} \to K^{*+}K^{*-}$	—	—	$0.097^{+0.35+0.43}_{-0.72-0.074}$	—
$\chi_{c1} \to \omega\omega$	—	—	$0.024^{+0.089+0.102}_{-0.017-0.020}$	—
$\chi_{c1} \to \phi\phi$	—	—	$0.110^{+0.358+0.476}_{-0.081-0.088}$	—
$\chi_{c2} \to \rho^0\rho^0$	$0.56^{+1.79+0.59}_{-0.39-0.41}$	$0.029^{+0.174+0.271}_{-0.024-0.022}$	$0.11^{+0.42+0.65}_{-0.08-0.11}$	$2.93^{+11.42+2.62}_{-2.15-1.32}$
$\chi_{c2} \to \rho^+\rho^-$	$1.12^{+3.58+1.18}_{-0.78-0.82}$	$0.058^{+0.348+0.542}_{-0.048-0.044}$	$0.22^{+0.84+1.30}_{-0.16-0.22}$	$5.86^{+22.84+5.24}_{-4.30-2.64}$
$\chi_{c2} \to K^*\bar{K}^*$	$1.47^{+4.43+1.76}_{-1.04-1.24}$	$0.138^{+0.833+1.109}_{-0.112-0.108}$	$0.34^{+1.20+2.00}_{-0.25-0.25}$	$8.95^{+36.90+10.21}_{-6.66-4.49}$
$\chi_{c2} \to K^{*+}K^{*-}$	$1.47^{+4.43+1.76}_{-1.04-1.24}$	$0.138^{+0.833+1.109}_{-0.112-0.108}$	$0.34^{+1.20+2.00}_{-0.25-0.25}$	$8.95^{+36.90+10.21}_{-6.66-4.49}$
$\chi_{c2} \to \omega\omega$	$0.72^{+2.27+0.77}_{-0.51-0.53}$	$0.039^{+0.232+0.360}_{-0.031-0.022}$	$0.14^{+0.53+0.83}_{-0.10-0.10}$	$3.78^{+14.8+3.45}_{-2.78-1.74}$
$\chi_{c2} \to \phi\phi$	$1.15^{+3.10+1.70}_{-0.81-1.10}$	$0.156^{+0.938+1.090}_{-0.126-0.147}$	$0.25^{+0.80+1.55}_{-0.18-0.24}$	$8.52^{+35.46+10.58}_{-6.35-4.21}$

The polarized decay widths $\Gamma_{+,+}$ and $\Gamma_{+,0}$ come solely from the twist-3 contribution, which therefore do not exist in Ref.[18] where only the leading twist distribution is considered. The twist-3 is the leading contribution for $\chi_{Q1} \to VV$ processes. In the literature, the polarized decay width $\Gamma_{+,0}$ of χ_{Q1} from the contribution of QED correction had been taken into account [13], however we find the QED contribution is several orders less than what from QCD, and hence are negligible. From Tables II and III we notice that the twist-3 contributions $\Gamma_{+,+}$ and $\Gamma_{+,0}$ are prominent for polarized widths of

TABLE IV: The decay widths of $\chi_{QJ} \to PP$ processes.

		$\eta_c\eta_c$	K^+K^-	$\pi^+\pi^-$
$\Gamma(\chi_{bJ})$[eV]	χ_{b0}	$20.90^{+3.72+3.44}_{-3.11-3.24}$	$1.75^{+0.31+0.06}_{-0.27-0.06}$	$0.85^{+0.16+0.02}_{-0.13-0.02}$
	χ_{b2}	$4.17^{+0.74+0.69}_{-0.62-0.65}$	$0.35^{+0.06+0.01}_{-0.05-0.01}$	$0.17^{+0.03+0.00}_{-0.02-0.00}$
$\Gamma(\chi_{cJ})$[keV]	χ_{c0}		$2.95^{+7.59+0.77}_{-1.97-0.70}$	$1.53^{+4.01+0.30}_{-1.03-0.28}$
	χ_{c2}		$0.76^{+1.95+0.20}_{-0.51-0.18}$	$0.39^{+1.03+0.08}_{-0.26-0.07}$

$\chi_{b0}/\chi_{b2} \to J/\psi J/\psi$ processes. While for processes $\chi_{c0}/\chi_{c2} \to K^*\bar{K}^*$ and $\chi_{c0}/\chi_{c2} \to \phi\phi$, though the twist-3 contributions are not predominant, they are still important.

Notice that the higher twist contributions are suppressed by powers of $\frac{m_V}{m_Q}$, and in dealing with the infrared divergences appearing in the next-to-leading contributions in twist expansion, factors of $\ln(\frac{m_Q}{\Lambda_{QCD}})$, $\ln^2(\frac{m_Q}{\Lambda_{QCD}})$, or $\frac{m_Q}{\Lambda_{QCD}}$ are induced via the regularization procedure (18). Except for polarized decay width $\Gamma_{+,\,0}$, which comes from the interference of twist-2 and -3 terms and hence has the power suppression of $\frac{m_V}{m_Q}$ at the amplitude level, other polarized decay widths from twist-3 contributions all have power suppression of $(\frac{m_V}{m_Q})^2$ at the amplitude level as expected. The $\Gamma_{0,\,0}$ contains only $\ln(\frac{m_Q}{\Lambda_{QCD}})$ form, the $\Gamma_{+,\,0}$ contains both $\ln(\frac{m_Q}{\Lambda_{QCD}})$ and $\ln^2(\frac{m_Q}{\Lambda_{QCD}})$ forms, while $\Gamma_{+,\,+}$ and $\Gamma_{+,\,-}$ contain all three kinds of regularized forms. So, as $m_V \sim \Lambda_{QCD}$, the convergence in twist expansion is hold well by power counting.

Of the processes $\chi_{b0}/\chi_{b2} \to J/\psi J/\psi$, the twist expansion fails. For $\chi_{b0} \to J/\psi J/\psi$, the polarized decay width $\Gamma_{+,\,+}$, which comes merely from the twist-3 contribution, is even bigger than $\Gamma_{0,\,0}$, mainly the twist-2 contribution. For $\chi_{b2} \to J/\psi J/\psi$ process, the twist-3 contribution $\Gamma_{+,\,0}$ is also larger than $\Gamma_{0,\,0}$. This is not a big surprise, though $m_{J/\psi}$ is smaller than $m_{\chi_{bJ}}$, it is still much larger than Λ_{QCD} in comparison with other light mesons, which spoils the twist expansions. Hence, the perturbative calculation on $\chi_{bJ} \to J/\psi J/\psi$ in this work can be treated as a qualitative estimation.

To estimate the branching ratios we use the following expressions for the total bot-

tomonium decay widths [18, 34]:

$$\Gamma_{\chi_{b0}} = \frac{3C_F}{N_c}\pi\alpha_s^2\frac{\langle O_P^{bb}\rangle}{m_b^4} + \frac{n_f}{3}\pi\alpha_s^2\frac{\langle O_8\rangle}{m_b^2} = 0.68 \text{ MeV}, \tag{23}$$

$$\Gamma_{\chi_{b2}} = \frac{4C_F}{5N_c}\pi\alpha_s^2\frac{\langle O_P^{bb}\rangle}{m_b^4} + \frac{n_f}{3}\pi\alpha_s^2\frac{\langle O_8\rangle}{m_b^2} = 0.20 \text{ MeV}, \tag{24}$$

$$\begin{aligned}\Gamma\chi_{b1} &= \frac{C_F\alpha_s^3}{N_c}\left[\left(\frac{587}{54} - \frac{317}{288}\pi^2\right)C_A + \left(-\frac{16}{27} - \frac{4}{9}\ln\frac{\Lambda}{2m_b}\right)n_f\right]\frac{\langle O_P^{bb}\rangle}{m_b^4} + \frac{n_f}{3}\pi\alpha_s^2\frac{\langle O_8\rangle}{m_b^2} \\ &= 0.09 \text{ MeV}.\end{aligned} \tag{25}$$

The total decay widths for χ_{cJ} are obtained from PDG [33], i.e., $\Gamma_{\chi_{c0}} = 10.3\text{MeV}$, $\Gamma_{\chi_{c1}} = 0.86\text{MeV}$, and $\Gamma_{\chi_{c2}} = 1.97\text{MeV}$.

The branching ratios of different processes are listed in Table V. They are evaluated by taking the center value of each unpolarized width and divided by the total decay widths of χ_{QJ}.

TABLE V: The branching fractions of various processes of p-wave heavy quarkonium exclusive decays to double mesons.

	$\rho^0\rho^0$	$K^*\bar{K}^*$	$\omega\omega$	$\phi\phi$	$J/\psi J/\psi$
χ_{b0}	3.9×10^{-6}	9.3×10^{-6}	3.2×10^{-6}	5.9×10^{-6}	4.9×10^{-5}
χ_{b1}	4.8×10^{-8}	2.4×10^{-7}	4.8×10^{-8}	1.3×10^{-7}	2.1×10^{-5}
χ_{b2}	7.3×10^{-6}	2.0×10^{-5}	9.3×10^{-6}	1.8×10^{-5}	6.7×10^{-4}
χ_{c0}	4.4×10^{-4}	10.9×10^{-4}	3.6×10^{-4}	6.2×10^{-4}	
χ_{c1}	1.7×10^{-4}	4.5×10^{-4}	1.1×10^{-4}	5.1×10^{-4}	
χ_{c2}	35×10^{-4}	11×10^{-3}	45×10^{-4}	98×10^{-4}	
	$\eta_c\eta_c$	K^+K^-	$\pi^+\pi^-$		
χ_{b0}	3.1×10^{-5}	2.6×10^{-6}	1.3×10^{-6}		
χ_{b2}	2.1×10^{-5}	1.8×10^{-6}	8.5×10^{-7}		
χ_{c0}		2.9×10^{-4}	1.5×10^{-4}		
χ_{c2}		3.8×10^{-4}	1.9×10^{-4}		

For comparison, in Table VI we also present the total decay width of $\chi_{bJ} \to J/\psi J/\psi$ obtained in other calculations. The second and third columns give what we obtained in this work. The fourth column shows the NRQCD results of Ref.[13], including QED and relativistic corrections. The last column presents the calculation results in light-cone formalism up to twist-2 [18].

TABLE VI: The total width of $\chi_{bJ} \to J/\psi J/\psi$ process in various calculation methods.

[eV]	Γ[twist-2]	Γ[twist-3]	Γ-NRQCD[13]	Γ-LC twist-2[18]
$\chi_{b0} \to J/\psi J/\psi$	$21.69^{+3.32}_{-2.86}$	$33.05^{+7.65+209.27}_{-6.02-28.48}$	5.54	$79. \pm 3.1 \pm 25. \pm 31.$
$\chi_{b1} \to J/\psi J/\psi$	0	$2.04^{+0.44+42.32}_{-0.36-1.00}$	9.04×10^{-7}	—
$\chi_{b2} \to J/\psi J/\psi$	$25.84^{+4.72}_{-3.93}$	$134.51^{+31.91+458.64}_{-25.33-95.17}$	10.6	$270. \pm 41. \pm 93. \pm 110.$

From the results in Table VI we notice that the light cone formalism generally yields more than what from the NRQCD calculation for J/ψ production, and the results obtained in Ref.[13] are much smaller than ours, especially for the $\chi_{b1} \to J/\psi J/\psi$ process. Further experimental measurement on $\chi_{b1} \to J/\psi J/\psi$ process may provide test about these two mechanisms. On the other hand, within the light cone formalism the results obtain in Ref. [18] are much larger than ours. We have compared the analytical result of the twist-2 part, find that there is a redundant factor of two in Ref. [18]. Furthermore, the different choices of input parameters may also induce certain uncertainties.

The comparison of our calculation with experimental results is given in Table VII. We can see from the table that our results in general agree with the experiment in all $\chi_{cJ} \to VV$ processes within an order of magnitude, while the NRQCD calculation in Ref.[13] for $\chi_{c1} \to VV$ processes gives results several orders smaller than the BESIII measurements. Except for $\chi_{c2} \to K^+K^-$, the calculated χ_{c0}/χ_{c2} to $\pi\pi$ and K^+K^- decay widths are about an order smaller than the PDG[33] data. To be noted that of all our calculation results, the $\chi_{cJ} \to \phi\phi$ processes possess larger branching ratios than that of the $\chi_{cJ} \to \omega\omega$ processes, while experimental data do not tell so all the way. This is an open question leaving for further study, and could be partly understood as the

TABLE VII: Branch ratios of $\chi_{cJ} \to VV$ and $\chi_{cJ} \to PP$, from BESIII experiment [36], the PDG data [33], and our calculation.

	Br(twist-3)	BESIII [36]	PDG [33]
$\chi_{c0} \to \phi\phi$	$(6.2^{+19.8+21.6}_{-4.3-2.78}) \times 10^{-4}$	$(8.0 \pm 0.3 \pm 0.8) \times 10^{-4}$	$(9.2 \pm 1.9) \times 10^{-4}$
$\chi_{c1} \to \phi\phi$	$(5.1^{+16.7+22.1}_{-3.8-4.1}) \times 10^{-4}$	$(4.4 \pm 0.3 \pm 0.5) \times 10^{-4}$	—
$\chi_{c2} \to \phi\phi$	$(98^{+402+159}_{-73-54}) \times 10^{-4}$	$(10.7 \pm 0.7 \pm 1.2) \times 10^{-4}$	$(14.8 \pm 2.8) \times 10^{-4}$
$\chi_{c0} \to \omega\omega$	$(3.6^{+10.4+6.0}_{-2.4-1.2}) \times 10^{-4}$	$(9.5 \pm 0.3 \pm 1.1) \times 10^{-4}$	$(22 \pm 7) \times 10^{-4}$
$\chi_{c1} \to \omega\omega$	$(1.1^{+4.1+4.7}_{-0.8-0.9}) \times 10^{-4}$	$(6.0 \pm 0.3 \pm 0.7) \times 10^{-4}$	—
$\chi_{c2} \to \omega\omega$	$(45^{+174+59}_{-33-23}) \times 10^{-4}$	$(8.9 \pm 0.3 \pm 1.1) \times 10^{-4}$	$(19 \pm 6) \times 10^{-4}$
$\chi_{c0} \to K^*\bar{K}^*$	$(10.9^{+34.0+26.4}_{-7.6-5.0}) \times 10^{-4}$	—	$(17 \pm 6 \pm 1) \times 10^{-4}$
$\chi_{c0} \to K^+K^-$	$(2.9^{+7.7+1.9}_{-0.7-0.7}) \times 10^{-4}$	—	$(6.10 \pm 0.35) \times 10^{-3}$
$\chi_{c0} \to \pi\pi$	$(1.5^{+3.9+0.3}_{-1.0-0.3}) \times 10^{-4}$	—	$(8.4 \pm 0.4) \times 10^{-3}$
$\chi_{c2} \to K^*\bar{K}^*$	$(10.6^{+43.0+16.4}_{-7.9-5.8}) \times 10^{-3}$	—	$(2.5 \pm 0.5) \times 10^{-3}$
$\chi_{c2} \to K^+K^-$	$(3.8^{+9.9+1.0}_{-2.5-0.9}) \times 10^{-4}$	—	$(10.9 \pm 0.8) \times 10^{-4}$
$\chi_{c2} \to \pi\pi$	$(2.0^{+5.2+0.4}_{-1.3-0.3}) \times 10^{-4}$	—	$(2.39 \pm 0.14) \times 10^{-3}$

difference in the selection of nonperturbative parameters ρ and ϕ in various χ_{QJ} decay processes.

IV. SUMMARY AND CONCLUSIONS

In this work, various exclusive processes of p-wave Quarkonium decays to two mesons are investigated, where the final state vector mesons in different polarizations are considered separately. For light mesons, we expand the light cone distribution amplitudes up to twist-3, which induces certain end-point singularities in the calculation. To hurdle these singularities, the prescription used in dealing with the B meson decay problems is employed, in which two complex parameters are introduced. Though these parameters induce some uncertainties in our calculation, we believe our result should be reliable as

an order estimation.

The χ_{c1} and χ_{b1} to VV decay processes are induced entirely by the twist-3 contribution in distribution amplitude expansion, which are firstly estimated in this work. It turns out that the higher twist contribution is significant and provides a possible explanation for the observation of hadron helicity selection rule violated processes $\chi_{c1} \to \phi\phi$ and $\omega\omega$ observed by the BESIII collaboration in recently. We also evaluate the $\chi_{b1} \to J/\psi J/\psi$ process and find that its branching ratio is big enough to be measured at the LHC and B-factory experiments.

Note that after this work has been done, the Belle Collaboration releases new measurement results on the p-wave spin-triplet bottomonium decays to double charmonium [12]. Their obtained upper limits of the branch ratios $Br(\chi_{b0} \to J/\psi J/\psi) < 7.1 \times 10^{-5}$, $Br(\chi_{b1} \to J/\psi J/\psi) < 2.7 \times 10^{-5}$, and $Br(\chi_{b2} \to J/\psi J/\psi) < 4.5 \times 10^{-5}$ at the 90% confidence level are compatible with our calculation, that is: $Br(\chi_{b0} \to J/\psi J/\psi) = (0.7 \sim 35.9) \times 10^{-5}$, $Br(\chi_{b1} \to J/\psi J/\psi) = (0.7 \sim 44) \times 10^{-5}$, and $Br(\chi_{b2} \to J/\psi J/\psi) = (6.9 \sim 295) \times 10^{-5}$, while taking the uncertainties in both theory and experiment into account.

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Appendix A: tensor structure of the amplitude

For vector mesons in the final state, the tensor $\mathcal{B}^{\mu\nu}$ reads:

$$\mathcal{B}^{\mu\nu} \;=\; T^0(\mathbf{B}^{\mu\nu} + \mathbf{C}^{\mu\nu} + \mathbf{D}^{\mu\nu} + \mathbf{S}^{\mu\nu}) \; , \tag{A1}$$

$$\begin{aligned}
\mathbf{B}^{\mu\nu} \;&= \frac{{f_V^T}^2 m_V^4 \varepsilon_1\cdot n_+ \varepsilon_2\cdot n_-}{{m_Q}^4}\{B_{k1}g^{\mu\nu} + B_{k2}(n_-^\mu n_-^\nu + n_-^\mu n_-^\nu) + B_{k3}(n_-^\mu n_+^\nu + n_+^\mu n_-^\nu) \\
&+B_{k4}g_\perp^{\mu\nu}\} + \frac{{f_V^T}^2 m_V^2}{m_Q^2}\{B_{k5}[\varepsilon_{\perp 1}^\mu(n_-^\nu - n_+^\nu)\varepsilon_2\cdot n_- + \varepsilon_{\perp 2}^\mu(n_+^\nu - n_-^\nu)\varepsilon_1\cdot n_-] \\
&+B_{k6}(n_-^\mu\varepsilon_{\perp 1}^\nu\varepsilon_2\cdot n_- + n_+^\mu\varepsilon_{\perp 2}^\nu\varepsilon_1\cdot n_+) + B_{k7}(n_+^\mu\varepsilon_{\perp 1}^\nu\varepsilon_2\cdot n_- + n_-^\mu\varepsilon_{\perp 2}^\nu\varepsilon_1\cdot n_+)\} \\
&+{f_V^T}^2\{B_{k8}(\varepsilon_{\perp 1}^\mu\varepsilon_{\perp 2}^\nu + \varepsilon_{\perp 1}^\nu\varepsilon_{\perp 2}^\mu) + B_{k9}\varepsilon_{\perp 1}\cdot\varepsilon_{\perp 2}\; g_\perp^{\mu\nu}\} \; ,
\end{aligned} \tag{A2}$$

$$\begin{aligned}
\mathbf{C}^{\mu\nu} \;&= \frac{f_V^2 m_V^2}{m_Q^2}\{C_{k1}(\varepsilon_{\perp 1}^\mu\varepsilon_{\perp 2}^\nu + \varepsilon_{\perp 1}^\nu\varepsilon_{\perp 2}^\mu) + C_{k2}(\varepsilon_{\perp 1}\cdot\varepsilon_{\perp 2})g^{\mu\nu} \\
&+C_{k3}(n_-^\mu n_-^\nu + n_+^\mu n_+^\nu)\varepsilon_{\perp 1}\cdot\varepsilon_{\perp 2} + C_{k4}(n_+^\mu n_-^\nu + n_-^\mu n_+^\nu)\varepsilon_{\perp 1}\cdot\varepsilon_{\perp 2} \\
&+C_{k5}[\varepsilon_{\perp 1}^\mu(n_-^\nu - n_+^\nu)\varepsilon_2\cdot n_- + \varepsilon_{\perp 2}^\mu(n_+^\nu - n_-^\nu)\varepsilon_1\cdot n_+] \\
&+C_{k6}(n_-^\mu\varepsilon_{\perp 1}^\nu\varepsilon_2\cdot n_- + n_+^\mu\varepsilon_{\perp 2}^\nu\varepsilon_1\cdot n_+) + C_{k7}(n_+^\mu\varepsilon_{\perp 1}^\nu\varepsilon_2\cdot n_- + n_-^\mu\varepsilon_{\perp 2}^\nu\varepsilon_1\cdot n_+) \\
&+C_{k8}\varepsilon_1\cdot n_+\varepsilon_2\cdot n_- g^{\mu\nu} + C_{k9}\varepsilon_1\cdot n_+\varepsilon_2\cdot n_-(n_-^\mu n_-^\nu + n_+^\mu n_+^\nu) \\
&+C_{k10}\varepsilon_1\cdot n_+\varepsilon_2\cdot n_-(n_+^\mu n_-^\nu + n_-^\mu n_+^\nu)\} \; ,
\end{aligned} \tag{A3}$$

$$\begin{aligned}
\mathbf{D}^{\mu\nu} \;&= \frac{\tilde{f}_V^2 m_V^2}{m_Q^2}\{D_{k1}(\varepsilon_{\perp 1}^\mu\varepsilon_{\perp 2}^\nu + \varepsilon_{\perp 1}^\nu\varepsilon_{\perp 2}^\mu) + D_{k2}\varepsilon_{\perp 1}\cdot\varepsilon_{\perp 2}g_\perp^{\mu\nu} + D_{k3}(\varepsilon_{\perp 1}\cdot\varepsilon_{\perp 2})g^{\mu\nu} \\
&+D_{k4}\varepsilon_{\perp 1}\cdot\varepsilon_{\perp 2}(n_+^\mu n_+^\nu + n_-^\mu n_-^\nu) + D_{k5}\varepsilon_{\perp 1}\cdot\varepsilon_{\perp 2}(n_+^\mu n_-^\nu + n_-^\mu n_+^\nu)\} \; ,
\end{aligned} \tag{A4}$$

$$\begin{aligned}
\mathbf{S}^{\mu\nu} \;&= \frac{-\tilde{f}_V f_V m_V^2}{m_Q^2}\{S_{k1}(\varepsilon_{\perp 1}^\mu\varepsilon_{\perp 2}^\nu + \varepsilon_{\perp 1}^\nu\varepsilon_{\perp 2}^\mu) + S_{k2}(\varepsilon_{\perp 1}^\mu\varepsilon_{\perp 2}^\nu + \varepsilon_{\perp 1}^\nu\varepsilon_{\perp 2}^\mu) \\
&+S_{k3}\varepsilon_{\perp 1}\cdot\varepsilon_{\perp 2}\; g_\perp^{\mu\nu} + S_{k4}\varepsilon_{\perp 1}\cdot\varepsilon_{\perp 2}(n_-^\mu n_-^\nu + n_+^\mu n_+^\nu - n_-^\mu n_+^\nu - n_+^\mu n_-^\nu) \\
&+S_{k5}[\varepsilon_1\cdot n_+(n_-^\nu - n_+^\nu)\varepsilon_{\perp 2}^\mu + \varepsilon_2\cdot n_-(n_+^\nu - n_-^\nu)\varepsilon_{\perp 1}^\mu] \\
&+S_{k6}(n_+^\mu\varepsilon_1^\nu\varepsilon_2\cdot n_- + n_-^\mu\varepsilon_2^\nu\varepsilon_1\cdot n_+) + S_{k7}(n_-^\mu\varepsilon_1^\nu\varepsilon_2\cdot n_- + n_+^\mu\varepsilon_2^\nu\varepsilon_1\cdot n_+)\} \; .
\end{aligned} \tag{A5}$$

Here, $T^0 = 2\frac{N_c^2-1}{16\sqrt{2}N_c^2\sqrt{N_c}m_Q^3}$ and $g_\perp^{\mu\nu} = g^{\mu\nu} - \frac{n_-^\mu n_+^\nu}{2} - \frac{n_+^\mu n_-^\nu}{2}$ comes from the derivative with respect to the transverse momentum, such as $\frac{\partial k_\perp^\mu}{\partial k_{\perp\nu}} = g_\perp^{\mu\nu}$.

The coefficients in above expressions are:

$$B_{k1} = -\frac{9}{8}\{(2(con^2-1)X_a^2 + 2\ln(2)(1+con^2+(1-con^2)X_l) - 4X_a + \pi^2\} \ , \tag{A6}$$

$$\begin{aligned} B_{k2} &= -\frac{9}{32}\{2(X_a^2 + (2+4con-2con^2)X_a - \ln(2)X_l - (1+\ln(2) \\ &\quad -4con\ln(2) + 2con^2\ln(2))) + (con^2-2con-1)\pi^2\} \ , \end{aligned} \tag{A7}$$

$$\begin{aligned} B_{k3} &= \frac{1}{128}\{72\ln(2)X_l - 72X_a^2 - 144(1-con)^2X_a + (72-288con \\ &\quad +144con^2)\ln(2) - 1 + 36(1-con)^2\pi^2\} \ , \end{aligned} \tag{A8}$$

$$\begin{aligned} B_{k4} &= \frac{9}{8}\{2(con-1)^2X_a^2 - 4(con+2)X_a + 2(1-con)\ln(2)X_l \\ &\quad +6(1+con)\ln(2)\} \ , \end{aligned} \tag{A9}$$

$$B_{k5} = \frac{9}{16}(1-con)(2X_a^2-\pi^2) \ , \tag{A10}$$

$$\begin{aligned} B_{k6} &= -\frac{9}{16}\{2(conX_a^2 - 4(1+con)X_a - 2(1+con)(X_l-5)\ln(2)) \\ &\quad +(3con+4)\pi^2\} \ , \end{aligned} \tag{A11}$$

$$\begin{aligned} B_{k7} &= \frac{9}{16}\{con(-2X_a^2 - 8X_a + 4(X_l+1)\ln(2) + \pi^2) - 8X_a \\ &\quad +4(X_l+1)\ln(2)\} \ , \end{aligned} \tag{A12}$$

$$B_{k8} = -\frac{9\pi^2}{2} \ , \tag{A13}$$

$$B_{k9} = \frac{9\pi^2}{2} \ , \tag{A14}$$

$$C_{k1} = -\frac{9}{16}\{10X_a^2 - 8X_a + 2\ln(2)X_l + 5\pi^2 + 6\ln(2)\} \ , \tag{A15}$$

$$C_{k2} = \frac{9}{8}\{6X_a^2 - 2X_a + \pi^2 - 2\ln(2)\}\ , \tag{A16}$$

$$C_{k3} = \frac{9}{64}\{-2X_a^2 + 2X_a - 4\ln(2)X_l + 3\pi^2 - 44 + 2\ln(2)\}\ , \tag{A17}$$

$$C_{k4} = -\frac{9}{64}\{10X_a^2 - 6X_a - 4\ln(2)X_l - 44 - 2\ln(2) + 5\pi^2\}\ , \tag{A18}$$

$$C_{k5} = \frac{9}{32}\{2X_a^2 + 3\pi^2 - 8\}\ , \tag{A19}$$

$$C_{k6} = -\frac{9}{8}\{2X_l\ln(2) - 4 - \ln(2)\}\ , \tag{A20}$$

$$C_{k7} = -\frac{9X_a}{2} - \frac{9\ln(2)X_l}{8} + \frac{45\ln(2)}{8}\ , \tag{A21}$$

$$C_{k8} = \frac{9\pi^2}{8}\ , \tag{A22}$$

$$C_{k9} = -\frac{9}{64}(\pi^2 - 16)\ , \tag{A23}$$

$$C_{k10} = -\frac{9}{64}(\pi^2 + 16)\ , \tag{A24}$$

$$D_{k1} = \frac{9}{8}\{2X_a^2 - 2X_a - \ln(2)X_l + 3\ln(2)\}\ , \tag{A25}$$

$$D_{k2} = \frac{9}{4}\{X_l - 1\}\ln(2)\ , \tag{A26}$$

$$D_{k3} = -\frac{9}{4}\{X_a^2 - X_a + \ln(2)\}\ , \tag{A27}$$

$$D_{k4} = -\frac{9}{32}\{X_a^2 + X_a - 2\ln(2)X_l - 2 + \ln(2)\}\ , \tag{A28}$$

$$D_{k5} = \frac{9}{32}\{7X_a^2 - 5X_a - 2X_l \ln(2) - 2 + 7\ln(2)\} \ , \tag{A29}$$

$$S_{k1} = \frac{9}{16}\{4\ln(2)X_l - 2X_a^2 - 4X_a + \pi^2 + 4\} \ , \tag{A30}$$

$$S_{k2} = -\frac{9}{8}\{-8X_a + \pi^2 + 2 + 8\ln(2)\} \ , \tag{A31}$$

$$S_{k3} = -\frac{9}{4}\{4X_a + (X_l - 5)\ln(2)\} \ , \tag{A32}$$

$$S_{k4} = \frac{9}{32}\{6X_a^2 - 16X_a + \pi^2 + 16\ln(2)\} \ , \tag{A33}$$

$$S_{k5} = -\frac{9}{32}\{\pi^2 - 2X_a^2\} \ , \tag{A34}$$

$$S_{k6} = \frac{9}{32}\{2(X_a^2 - 4X_a - 2\ln(2)X_l + 6\ln(2)) + 3\pi^2\} \ , \tag{A35}$$

$$S_{k7} = -\frac{9}{32}\{4(X_l + 1)\ln(2) - X_a^2 - 8X_a + \pi^2\} \ , \tag{A36}$$

with $con = 1 - \frac{f_V(m_1+m_2)}{f_V^T m_V}$.

For pseudoscalar mesons in final state:

$$\mathcal{B}^{\mu\nu} = T^0 \mathbf{B}^{\mu\nu} \ , \tag{A37}$$

$$\begin{aligned}\mathbf{B}^{\mu\nu} = \ & f_P^2\{\frac{\mu_P^2}{m_Q^2}[(M_{k1}g_\perp^{\mu\nu}) + M_{k2}g^{\mu\nu} + M_{k3}(n_-^\mu n_-^\nu + n_+^\mu n_+^\nu) + M_{k4}(n_-^\mu n_+^\nu + n_+^\mu n_-^\nu)] \\ & + M_{k5}g_{\mu\nu} + M_{k6}(n_-^\mu n_-^\nu + n_+^\mu n_+^\nu) + M_{k7}(n_-^\mu n_+^\nu + n_+^\mu n_-^\nu)\}.\end{aligned} \tag{A38}$$

Here T^0 is the same as the vector meson case in above, and

$$M_{k1} = 6\{\ln(2) - X_a\} \ , \tag{A39}$$

$$M_{k2} = 2\{X_a - \ln(2)\} \ , \tag{A40}$$

$$M_{k3} = \frac{1}{8}\{2X_a^2 - 8X_a + 2X_l\ln(2) + 1 + 6\ln(2)\} \ , \tag{A41}$$

$$M_{k4} = \frac{1}{4}\{X_a^2 + (X_l - 1)\ln(2)\} \ , \tag{A42}$$

$$M_{k5} = \frac{9\pi^2}{2} \ , \tag{A43}$$

$$M_{k6} = 9 - \frac{9\pi^2}{16} \ , \tag{A44}$$

$$M_{k7} = -\frac{9}{16}\{\pi^2 + 16\} \ . \tag{A45}$$

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